第 1 题

英文原题

Helen's vinyl record collection contains 33 LP records and 45 EP records. An LP record plays for three times as long as an EP record. How many more EP records must Helen buy so that the total playing time of her EP records equals the total playing time of her LP records?

(A) 45 (B) 54 (C) 63 (D) 78 (E) 87

中文翻译

Helen 收藏了 33 张 LP 黑胶唱片和 45 张 EP 黑胶唱片. 每张 LP 唱片的播放时长是每张 EP 唱片的 3 倍. Helen 还需要购买多少张 EP 唱片, 才能使全部 EP 唱片的总播放时长等于全部 LP 唱片的总播放时长?

(A) 45 (B) 54 (C) 63 (D) 78 (E) 87

第 2 题

英文原题

A fruit punch is made by mixing a concentrated syrup with water. The weight of the syrup is 32% of the weight of the water. To the nearest whole number, what percentage of the total weight of the punch is the syrup?

(A) 24% (B) 28% (C) 32% (D) 68% (E) 76%

中文翻译

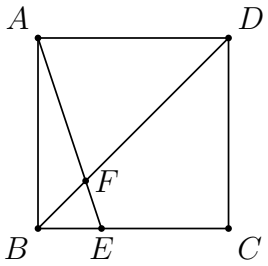
一种果汁饮料由浓缩糖浆和水混合而成. 糖浆的质量为水的质量的 32%. 糖浆占饮料总质量的百分比, 四舍五入到最接近的整数, 为多少?

(A) 24% (B) 28% (C) 32% (D) 68% (E) 76%

第 3 题

英文原题

Square $ABCD$ has side length 24. Let E lie on BC with $BE = 8$, and let F be the intersection of AE and BD.



What is the area of $\triangle BEF$?

(A) 24 (B) 32 (C) 45 (D) 48 (E) 62

中文翻译

正方形 $ABCD$ 的边长为 24. 点 E 在边 BC 上，且 $BE = 8$，点 F 为 AE 与 BD 的交点. 求 $\triangle BEF$ 的面积.

(A) 24 (B) 32 (C) 45 (D) 48 (E) 62

第 4 题

英文原题

Half of the internal angles of a polygon measure 150° each, and the other half of its internal angles measure 200° each. How many sides does the polygon have?

(A) 36 (B) 40 (C) 60 (D) 72 (E) 180

中文翻译

一个多边形有一半的内角各为 150°，另一半的内角各为 200°. 这个多边形有多少条边?

(A) 36 (B) 40 (C) 60 (D) 72 (E) 180

第 5 题

英文原题

A sequence of real numbers $a_1, a_2, a_3, \ldots$ is defined by $a_1 = 3$, and for all $n \geq 1$, $a_{n+1} = a_n + 2n + 1$. What is the value of a_{10}?

(A) 100 (B) 101 (C) 102 (D) 105 (E) 120

中文翻译

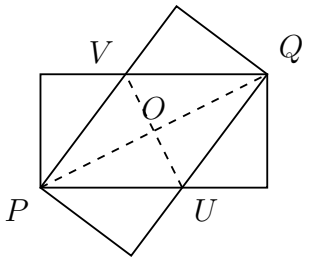
实数列 $a_1, a_2, a_3, \ldots$ 满足 $a_1 = 3$，且对所有 $n \geq 1$，有 $a_{n+1} = a_n + 2n + 1$. 求 a_{10}.

(A) 100 (B) 101 (C) 102 (D) 105 (E) 120

第 6 题

英文原题

Two congruent rectangles, each with side lengths 2 and 1, are placed so that they share a diagonal, as shown in the diagram. The area of the octagon formed is m/n, where m and n are relatively prime positive integers. Find $m + n$.



(A) 13 (B) 14 (C) 15 (D) 16 (E) 18

中文翻译

两个全等矩形的边长均为 2 和 1，按图摆放，共用一条对角线. 它们组成的八边形面积为 $\frac{m}{n}$，其中 m, n 为互质正整数. 求 $m + n$.

(A) 13 (B) 14 (C) 15 (D) 16 (E) 18

第 7 题

英文原题

How many ordered triples of positive integers (x, y, z) satisfy both of the following equations?

$$x + y + z = 31$$
$$x + 3y + 3z = 43$$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

中文翻译

有多少个正整数有序三元组 (x, y, z) 同时满足以下两个方程?

$$\begin{cases} x + y + z = 31, \\ x + 3y + 3z = 43. \end{cases}$$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

第 8 题

英文原题

A driver sees an electronic highway sign stating that the next exit is 8 miles away and it will take 5 minutes to reach it. Assume both numbers have been rounded to the nearest integer. Among the following options, what is the lowest speed limit (in miles per hour) that would possibly allow the driver to legally reach the exit within the given time frame?

(A) 65 (B) 70 (C) 75 (D) 80 (E) 85

中文翻译

一位司机看到高速公路电子路牌显示：下一个出口距此 8 英里，预计需要 5 分钟到达. 假设这两个数都已四舍五入到最接近的整数. 在下列选项中，哪一个是可能允许司机在所示时间范围内合法到达出口的最低限速？单位为英里/小时.

(A) 65 (B) 70 (C) 75 (D) 80 (E) 85

第 9 题

英文原题

Two similar but not congruent scalene triangles each have a side with length 9 and a side with length 12. What is the length of the longest side of the larger triangle?

(A) 12 (B) 14 (C) 15 (D) 16 (E) 17

中文翻译

两个相似但不全等的不等边三角形，都有一条边长为 9，另一条边长为 12. 较大三角形的最长边长是多少?

(A) 12 (B) 14 (C) 15 (D) 16 (E) 17

第 10 题

英文原题

For what odd positive integer n is the mean of $1, 4, 9, 16, \ldots, n^2$ exactly 30% greater than its median?

(A) 19 (B) 39 (C) 49 (D) 89 (E) 169

中文翻译

对于哪个奇正整数 n，数列 $1, 4, 9, 16, \ldots, n^2$ 的平均数恰好比其中位数大 30%?

(A) 19 (B) 39 (C) 49 (D) 89 (E) 169

第 11 题

英文原题

Gary writes down six distinct positive integers: $2, 5, 24, 60, 240$, and a three-digit number n. One of those six numbers is the least common multiple of all six of them. What is the sum of all possible values of n?

(A) 960 (B) 1680 (C) 2160 (D) 2280 (E) 2400

中文翻译

Gary 写下六个互不相同的正整数：$2, 5, 24, 60, 240$，以及一个三位数 n．这六个数中的某一个数，恰好是全部六个数的最小公倍数．求 n 所有可能取值之和．

（A）960 （B）1680 （C）2160 （D）2280 （E）2400

第 12 题

英文原题

A drawer has 100 red socks and 100 blue socks. Pippi randomly removes one sock at a time, without replacement, until all the socks of one color have been removed. What is the probability that there are exactly 2 socks left in the drawer once that happens?

（A）$\frac{27}{196}$ （B）$\frac{49}{198}$ （C）$\frac{247}{990}$ （D）$\frac{50}{199}$ （E）$\frac{25}{99}$

中文翻译

抽屉中有 100 只红袜子和 100 只蓝袜子．Pippi 每次随机取出一只袜子，取出后不放回，直到某一种颜色的袜子全部被取出为止．停止时，抽屉中恰好剩下 2 只袜子的概率是多少?

（A）$\frac{27}{196}$ （B）$\frac{49}{198}$ （C）$\frac{247}{990}$ （D）$\frac{50}{199}$ （E）$\frac{25}{99}$

第 13 题

英文原题

Two bicycle clubs, with disjoint memberships, go on a tour together. On the morning of the tour, members greet each other with a handshake. Everyone shakes hands with everyone else exactly once. There were 114 handshakes between members of the same club and 117 handshakes between members of different clubs. How many more people came from the larger club than from the smaller club?

（A）0 （B）4 （C）6 （D）7 （E）8

中文翻译

两个自行车俱乐部一同出游，没有人同时属于两个俱乐部．出游当天早上，所有人互相握手，每两个人恰好握手一次．同一俱乐部成员之间共握手 114 次，不同俱乐部成员之间共握手 117 次．人数较多的俱乐部比人数较少的俱乐部多来了多少人?

（A）0 （B）4 （C）6 （D）7 （E）8

第 14 题

英文原题

Consider a triangle with positive area whose side lengths are functions of t given by $a = t^2 + 1$, $b = 5t + 7$, and $c = 4t + 4$. How many different integer values of t are possible?

(A) 0 (B) 7 (C) 9 (D) 10 (E) infinitely many

中文翻译

一个面积为正的三角形，其三边长是变量 t 的函数：

$$a = t^2 + 1, \qquad b = 5t + 7, \qquad c = 4t + 4.$$

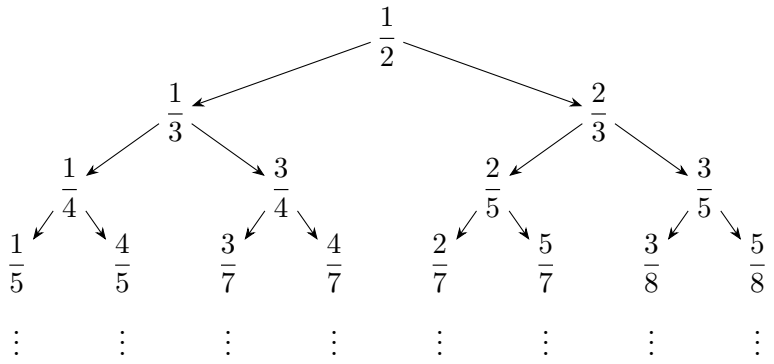
变量 t 有多少个不同的整数取值?

(A) 0 (B) 7 (C) 9 (D) 10 (E) 无穷多个

第 15 题

英文原题

The tree diagram below has nodes labeled with rational numbers. The top node, at level 0, is $\frac{1}{2}$. Each node $\frac{a}{b}$ sprouts two nodes in the level below it, $\frac{a}{a+b}$ and $\frac{b}{a+b}$. For example, level 1 has nodes $\frac{1}{1+2} = \frac{1}{3}$ and $\frac{2}{1+2} = \frac{2}{3}$.



At what level does the node $\frac{20}{27}$ appear in the diagram?

(A) 5 (B) 8 (C) 12 (D) 26 (E) The node $\frac{20}{27}$ does not appear in the diagram.

中文翻译

图示树状图的节点标注着有理数. 最上方的节点位于第 0 层，标注为 $\frac{1}{2}$. 每个标注为 $\frac{a}{b}$ 的节点，在下一层产生两个子节点，分别标注为 $\frac{a}{a+b}$ 和 $\frac{b}{a+b}$. 例如，第 1 层的两个节点为 $\frac{1}{3}$ 与 $\frac{2}{3}$. 节点 $\frac{20}{27}$ 出现在第几层?

(A) 5 (B) 8 (C) 12 (D) 26 (E) 该树状图中不出现节点 $\frac{20}{27}$

第 16 题

英文原题

A mouse starts in the center cell of a 3×3 grid. Every 6 seconds, it moves to a cell sharing an edge with its current cell, choosing each neighboring cell with equal probability. What is the expected number of seconds it takes for the mouse to first return to the center cell?

(A) 24 (B) 30 (C) 32 (D) 36 (E) 45

中文翻译

一只老鼠从 3×3 方格的中心格出发. 每隔 6 秒，它等概率地移到一个与当前格共边的相邻格. 它首次回到中心格所需秒数的期望是多少?

(A) 24 (B) 30 (C) 32 (D) 36 (E) 45

第 17 题

英文原题

A triangle with side lengths 18, 42, and 48 is inscribed in a circle, dividing the interior of the circle into four regions. Let A, B, and C be the areas of the three non-triangular regions, where $A<B<C$. Find the value of $A+C$.

(A) $392\pi-69\sqrt{3}$

(B) $392\pi-117\sqrt{3}$

(C) $196\pi-69\sqrt{3}$

(D) $392\pi+69\sqrt{3}$

(E) $588\pi-69\sqrt{3}$

中文翻译

一个三边长分别为 $18, 42, 48$ 的三角形内接于圆，把圆的内部划分为四个区域. 三个非三角形区域的面积分别为 A, B, C，且 $A < B < C$. 求 $A + C$.

(A) $392\pi - 69\sqrt{3}$

(B) $392\pi - 117\sqrt{3}$

(C) $196\pi - 69\sqrt{3}$

(D) $392\pi + 69\sqrt{3}$

(E) $588\pi - 69\sqrt{3}$

第 18 题

英文原题

The strictly decreasing sequence of 11 positive integers $(a_0, a_1, a_2, \ldots, a_{10})$ is defined by the recursive rule $a_k = \frac{1}{2}a_{k-1} + 6$ for $k \geq 1$. Let N be the minimum possible value of a_0. What is the sum of the digits of N?

(A) 7 (B) 9 (C) 10 (D) 11 (E) 14

中文翻译

严格递减的 11 个正整数组成的数列 $(a_0, a_1, \ldots, a_{10})$ 满足递推关系

$$a_k = \frac{1}{2}a_{k-1} + 6 \qquad (k \geq 1).$$

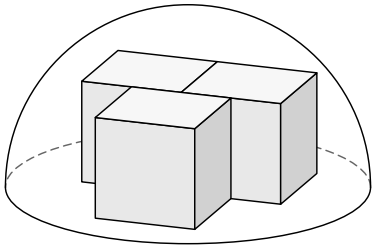
设 N 为 a_0 的最小可能值，求 N 的各位数字之和.

(A) 7 (B) 9 (C) 10 (D) 11 (E) 14

第 19 题

英文原题

Three $2 \times 2 \times 2$ cubes sit on a table so that two of the cubes share a face, and a face of the third cube is centered along a shared edge of the other two cubes, as shown.



A hemispherical cover is placed over the cubes so that its circumference rests on the table and the cubes are under the cover. What is the least possible value of the radius of the cover?

(A) 3 (B) $\frac{\sqrt{357}}{6}$ (C) $\frac{\sqrt{681}}{8}$ (D) $\frac{\sqrt{178}}{4}$ (E) $2\sqrt{3}$

中文翻译

三个 $2 \times 2 \times 2$ 的正方体放在桌面上. 其中两个正方体共用一个面，第三个正方体的一个面的竖直中线与前两个正方体的一条公共棱重合，如图所示. 将一个半球形罩盖在这三个正方体上，使半球的底面圆周落在桌面上，且三个正方体都被罩住. 求半球罩半径的最小可能值.

(A) 3 (B) $\frac{\sqrt{357}}{6}$ (C) $\frac{\sqrt{681}}{8}$ (D) $\frac{\sqrt{178}}{4}$ (E) $2\sqrt{3}$

第 20 题

英文原题

Boxes with dimensions $77 \times 78 \times 85$ or $2 \times 3 \times 85085$ have volume 510,510. How many different boxes with integer side lengths, including these two, have volume 510,510? Boxes are considered different if they have different dimensions.

(A) 63 (B) 90 (C) 301 (D) 365 (E) 966

中文翻译

长方体的尺寸为 $77 \times 78 \times 85$ 或 $2 \times 3 \times 85085$ 时，体积均为 510510. 各边长均为整数、体积为 510510 的长方体（包括这两种），共有多少种不同的尺寸？交换边长次序不另计.

(A) 63 (B) 90 (C) 301 (D) 365 (E) 966

第 21 题

英文原题

Gerry is playing the Triples Game. Her task is to partition the 3×3 grid of squares shown on the left into three sections, each containing three adjoining squares. (Three squares are adjoining if each shares an edge with at least one of the other two.) One point will be awarded for each section containing more X's than O's. Shown on the right is a sample partition that will earn one point in total. A partition that earns two or more points will

win the game. What fraction of the possible partitions of this particular grid will result in a win for Gerry?

X	O	X
X	X	O
O	O	O

原方格

X	O	X
X	X	O
O	O	O

得 1 分的示例

(A) $\frac{3}{10}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

中文翻译

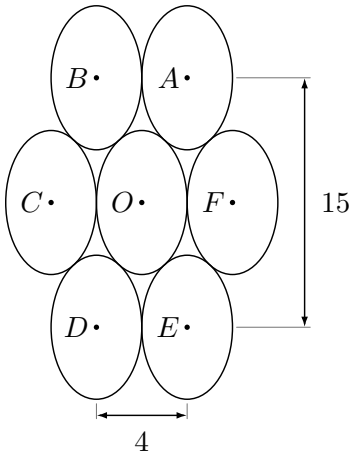
Gerry 正在玩“三格游戏”. 她需要把下列 3×3 方格分成三个区域，每个区域包含三个相邻小格. 这里，三个小格相邻是指每个小格都至少与另外两个中的一个共用一条边. 每个区域中，若 X 的个数多于 O 的个数，就得到 1 分. 图中右侧给出了一种总分为 1 的划分示例. 总分至少为 2 就获胜. 求这个方格的所有可能划分中，使 Gerry 获胜的划分所占的比例.

(A) $\frac{3}{10}$ (B) $\frac{1}{3}$ (C) $\frac{2}{5}$ (D) $\frac{4}{9}$ (E) $\frac{1}{2}$

第 22 题

英文原题

An engineer is designing a closely packed field of 7 wind turbines for electricity production. Each turbine, whose center is marked by a labeled dot in the figure, occupies one of 7 congruent elliptical regions. Turbines in the same row have centers that are 4 units apart, and turbines in the same column have centers that are 15 units apart, as shown in the figure below.



The major axis of each ellipse is $m\sqrt{p}$, where m and p are positive integers and p is not divisible by the square of any prime. What is $m+p$?

(A) 6 (B) 8 (C) 9 (D) 10 (E) 12

中文翻译

七台风力涡轮机分别占据七个全等、紧密相切的椭圆区域，椭圆中心如图标记. 同行相邻中心相距 4，同列上下中心相距 15. 每个椭圆的长轴长为 $m\sqrt{p}$，其中 m,p 为正整数，且 p 不被任何素数的平方整除. 求 $m+p$.

(A) 6 (B) 8 (C) 9 (D) 10 (E) 12

第 23 题

英文原题

In a list of one or more consecutive integers, there are p multiples of 2 and q multiples of 3. How many ordered pairs (p,q) are possible for this list if p and q are between 0 and 99, inclusive?

(A) 232 (B) 233 (C) 234 (D) 235 (E) 236

中文翻译

在一个由一个或多个连续整数组成的数列中，有 p 个 2 的倍数和 q 个 3 的倍数. 若 p,q 都在 0 到 99 之间，含端点，则可能的有序数对 (p,q) 共有多少个?

(A) 232 (B) 233 (C) 234 (D) 235 (E) 236

第 24 题

英文原题

How many polynomials $f(x)$ of degree 3 satisfying

$$f(2+\sqrt{5}) = f(2-\sqrt{5})$$

are there that have leading coefficient equal to 1 and all of whose coefficients are integers between -10 and 10, inclusive?

(A) 21 (B) 42 (C) 63 (D) 84 (E) 105

中文翻译

有多少个三次多项式 $f(x)$ 满足

$$f(2+\sqrt{5}) = f(2-\sqrt{5}),$$

且最高次项系数为 1，所有系数都是 -10 到 10 之间的整数，含端点?

(A) 21 (B) 42 (C) 63 (D) 84 (E) 105

第 25 题

英文原题

Chris the squirrel has 20 rooms in his cavern for storing food. The first room contains 1 nut, the second room contains 2 nuts, the third room contains 3 nuts, and so on, with the 20th room containing 20 nuts. Starting from today, Chris will collect 18 additional nuts each day and store them one per room in 18 distinct rooms in his cavern. His goal is to have all the rooms contain the same number of nuts once he is finished. What is the minimum number of days required to accomplish this goal?

(A) 75 (B) 95 (C) 105 (D) 190 (E) 195

中文翻译

松鼠 Chris 的洞穴里有 20 间储粮室. 第一间有 1 颗坚果，第二间有 2 颗，依此类推，第二十间有 20 颗. 从今天起，Chris 每天收集额外的 18 颗坚果，并选取 18 间不同的储粮室，每间放入 1 颗. 他的目标是在结束时使所有房间中的坚果数量相同. 至少需要多少天?

(A) 75 (B) 95 (C) 105 (D) 190 (E) 195